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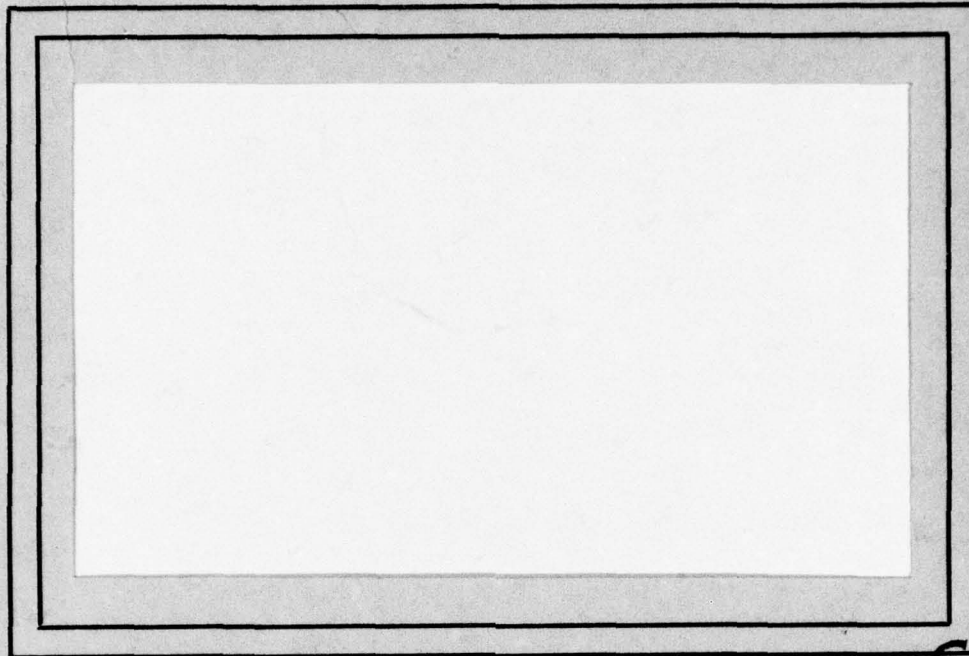


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LOG DIAMETER RESTRICTED

BOTTOM-UP TRIANGLE CELLULAR ACCEPTORS (UTCA's)

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#### ABSTRACT

The languages accepted by UTCA's in log diameter time are the same as those accepted by UTCA's in which a cell's new state depends only on its sons' states and not on its own preceding state. This set of languages remains the same if we allow log diameter + constant time, but it increases if we allow 2 log diameter time. It is also shown that this set is the same as the set of languages generated by a special class of "power of 2" OL-systems.

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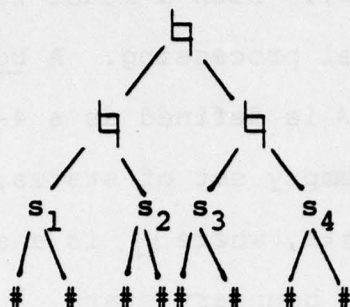
# 1. Basic concepts of UTCA's

The notion of a bottom-up triangle cellular acceptor was introduced by Dyer [1]. Such a model for computation is a special case of parallel processing. A bottom-up triangle cellular acceptor (UTCA)  $A$  is defined as a 4-tuple  $(S, I, F, \delta)$ , where  $S$  is a finite, nonempty set of states,  $I \subseteq S$  is a set of input states with  $\sqcup, \# \in I$ , where  $\sqcup$  is a special quiescent state,  $\#$  is a special boundary state,  $F \subseteq S$  is a set of accepting states, and  $\delta: S^3 \rightarrow S$  is called the state transition function satisfying  $\delta(s_1, s_2, s_3) = \#$  iff  $s_1 = \#$ , for arbitrary  $s_2, s_3$  in  $S$ , and  $\delta(\sqcup, \sqcup, \sqcup) = \sqcup$

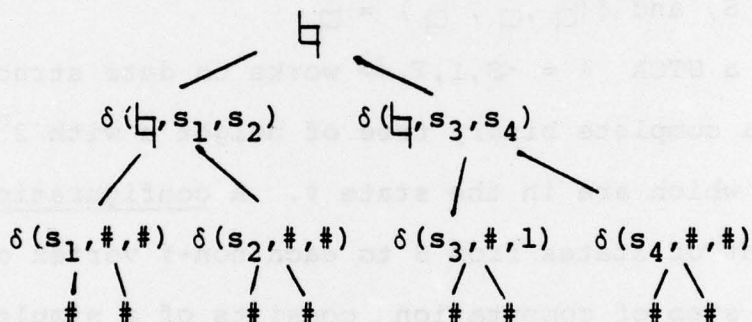
Such a UTCA  $A = \langle S, I, F, \delta \rangle$  works on data structures in the form of a complete binary tree of height  $n$  with  $2^{n+1}$  initial vertices which are in the state  $\#$ . A configuration of  $A$  is an assignment of states from  $S$  to each non- $\#$  vertex of such a tree. A step of computation consists of a simultaneous state transition at each non- $\#$  vertex of such a tree; if  $v_1, v_2$  denote the left and right son of a vertex  $v$  in this tree, and if in a certain configuration the vertices  $v, v_1, v_2$  are in the states  $s, s_1, s_2$ , respectively, then  $\delta(s, s_1, s_2)$  is the state of  $v$  in the next configuration. An input  $\sigma \in (I - \{\#\})^*$  of length  $2^n$  is initially in the  $2^n$  base (leaf) vertices of this tree, i.e., in each base vertex we have one symbol (state) of  $\sigma$ . In such an input configuration all other vertices of the tree are in

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the quiescent state  $\sqcup$ . The following figure shows an input configuration for the case  $n = 2$ .



For this case, after the first step of computation we have the following configuration:



An input string  $\sigma \in (I - \{\#\})^*$  is accepted by a UTCA  $A$  iff given the initial configuration defined by  $\sigma$ , the root of the tree enters an accepting state after some number of steps. Let  $L$  be a language over the alphabet  $I - \{\sqcup, \#\}$ . The language  $L$  is accepted by a UTCA  $A$  iff  $L$  is the set of all strings  $\sigma \in (I - \{\sqcup, \#\})^*$  such that  $\sigma \sqcup \dots \sqcup$ , where  $\text{length}(\sigma \sqcup \dots \sqcup) = 2^n$  for some  $n \geq 0$ , is accepted by  $A$ . A UTCA  $A$  is said to accept a language  $L$  within time  $T(|\sigma|)$  iff for any string  $\sigma$  in the language,  $A$  accepts the string within  $T(2^n)$  steps, where

$T: N \rightarrow N$ , and  $2^n$  is the smallest power of 2 equal to or greater than length  $(\sigma) = |\sigma|$ .  $T(|\sigma|) = \log |\sigma|$  is called log diameter time. Finally, let  $UTCA - TIME(T(|\sigma|))$  be the class of all languages which can be accepted by UTCA's within time  $T(|\sigma|)$ .

Example 1.  $L = \{a^m b^m \mid 2m \text{ is a power of } 2\}$  is not a regular language. The following UTCA  $A = \langle S, I, F, \delta \rangle$  accepts  $L$  within time  $\log |\sigma|$ :

$S = \{a, b, t, f, \sqcup, \#\}$ ,  $I = \{a, b, \sqcup, \#\}$ ,  $F = \{t\}$ , and  $\delta$  is specified as

<u>current state</u>	<u>left son's state</u>	<u>right son's state</u>	<u>next state</u>
$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$
$\sqcup$	a	a	a
$\sqcup$	b	b	b
$\sqcup$	a	b	t
all other combinations of states			f

A string  $\sigma \in \{a, b\}^*$  is accepted by  $A$  iff the states a, b meet together in the root, and not otherwise.

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## 2. Strong UTCA's

Strong bottom-up triangle cellular acceptors (SUTCA's) are a simplification of UTCA's in which the next state depends on the states of the two sons only, and not on the current state of the vertex. The formal definition of this SUTCA follows the UTCA definition; only the state transition function needs to be modified to  $\delta: S^2 \rightarrow S$ . Let  $L(\text{SUTCA})$  be the class of all languages which can be accepted by SUTCA's (in any time).

Theorem 1:  $L(\text{SUTCA}) = \text{UTCA} - \text{TIME}(\log |\sigma|)$

Proof:  $\subseteq$ : Let  $L$  be a language which is accepted by an SUTCA  $A = \langle S, I, F, \delta \rangle$ . Suppose that there exists a string  $\sigma \in L$  such that  $A$  accepts  $\sigma$  in more than  $\log |\sigma|$  steps. Thus, for the input configuration defined by  $\sigma$ , the first wave of non- $\sqcup$  states does not reach the root in a final state  $s \in F$ , but there exists a forthcoming wave of information from the base vertices to the root for which the root enters an accepting state. This forthcoming wave of information starts with base vertices which are all in the state  $\delta(\#, \#)$  already. Thus,  $L$  is either  $\emptyset$  or the set  $(I - \{\sqcup, \#\})^*$ . In both cases,  $L$  can be accepted by an SUTCA within time  $\log |\sigma|$ . We thus have the result that every language which can be accepted by an SUTCA can be accepted by an SUTCA within time  $\log |\sigma|$ , and thus by a UTCA within time  $\log |\sigma|$ .

$\supseteq$ : Let  $L$  be accepted by a UTCA  $A = \langle S, I, F, \delta \rangle$  within time  $\log |\sigma|$ . If  $\sigma \in L$  then  $A$  accepts  $\sigma$  after  $\log |\sigma|$  steps at most,

i.e., only the first wave of information from the base vertices to the root is relevant to whether a string will be accepted or not. Above layer 0 (base vertices) this first wave of information enters each vertex in state  $q$  only, according to our convention that  $\sigma(q, q, q) = q$ . Thus, we can restrict  $\delta(q, s_1, s_2) = s_3$  to  $\delta'(s_1, s_2, ) = s_3$ , i.e.,  $L$  can be accepted by an SUTCA.  $\square$

In our further considerations we will use Theorem 1 to simplify the definition of UTCA's which accept languages within time  $\log |\sigma|$ .

### 3. More time

A natural question is how much time may be required for a UTCA to accept a language which is not in the class  $UTCA - TIME(\log |\sigma|)$ .

Theorem 2. (a)  $UTCA - TIME(\log |\sigma|) = UTCA - TIME(\log |\sigma| + m)$ , for all  $m \geq 0$ .

(b)  $UTCA - TIME(\log |\sigma|) \subsetneq UTCA - TIME(2 \cdot \log |\sigma|)$ .

Proof: (a) Let  $L$  be accepted by a UTCA  $A = \langle S, I, F, \delta \rangle$  within time  $\log |\sigma| + m$ , for a certain  $m \geq 0$ . Then, a string  $\sigma$  is accepted by  $A$  iff at least the  $(m+1)$ st wave of information from the base vertices to the root transforms the root into an accepting state. During these  $m+1$  waves of information each vertex enters  $m+1$  states. We build up a new UTCA  $A' = \langle S', I, F', \delta' \rangle$  which accepts  $L$  within time  $\log |\sigma|$ . Without loss of generality, we assume that  $\delta(s, \#, \#) = s$ , for all  $s \in I$ . Now, let  $S' = S^{m+1}UI$ , and let  $\delta'$  be specified as follows:

Layer 1:

$\delta'(a, b) = (a_1, a_2, \dots, a_{m+1})$ , for  $a_1 = \delta(\sqcup, a, b)$ ,  $a_2 = \delta(a_1, a, b), \dots$ ,  $a_{m+1} = \delta(a_m, a, b)$ , and  $a, b \in I$ .

Layers  $k$ ,  $k \geq 2$ :

$\delta'((a_1, a_2, \dots, a_{m+1}), (b_1, b_2, \dots, b_{m+1})) = (c_1, c_2, \dots, c_{m+1})$ , for  $c_1 = \delta(\sqcup, a_1, b_1)$ ,  $c_2 = \delta(c_1, a_2, b_2), \dots, c_{m+1} = \delta(c_m, a_{m+1}, b_{m+1})$ .

Then,  $A'$  accepts a string  $\sigma$  iff given the initial configuration defined by  $\sigma$ , the root of  $A'$  enters a state  $(c_1, c_2, \dots, c_{m+1})$  such that at least one  $c_i$  is in  $F$ . The UTCA  $A'$  accepts the same language as  $A$  does.

(b) Let  $L = \{w \in \{a,b\}^* \mid \text{the number of } a\text{'s in } w \text{ is equal to or greater than the number of } b\text{'s in } w\}$ . According to [1],  $L$  can be accepted by a UTCA within time  $2 \cdot \log |\sigma|$ . Suppose that the UTCA  $A$  accepts  $L$  within time  $\log |\sigma|$ . In the tree of this UTCA, each vertex  $v$  in layer  $k$  computes at time step  $k+1$  a state  $s$  representing the states in  $v$ 's base at time step 0. Assume that at time step  $k+1$  for a vertex  $v$  in layer  $k+1$  the left son  $v'$  enters an accepting state, and the right son  $v''$  enters a non-accepting state. At time step  $k+2$  the vertex  $v$  must enter an accepting state iff the concatenation of the bases of  $v'$  and  $v''$  at time step 0 is in  $L$ . For this decision, the states of  $v'$  and  $v''$  at time step  $k+1$  must encode the differences between the numbers of  $a$ 's and  $b$ 's in their bases at time step 0. With respect to the finite number of states of  $A$ , and since  $k$  can be any natural number, such an encoding is impossible.  $\square$

This theorem can be used to speed up algorithms for UTCA's which operate within time  $\log |\sigma| + m$ , for  $m \geq 1$ . As an example, DYER shows in [1] that each regular language can be accepted by a UTCA within time  $\log |\sigma| + 1$ . According to Theorem 2(a) and Example 1, we have

$$\text{REG} \subsetneq \text{UTCA} - \text{TIME}(\log |\sigma|),$$

where REG denotes the class of all regular languages. For instance, the language of all strings of  $a$ 's and  $b$ 's in which

the b's are connected have at most one run of b's, is a regular language and can therefore be accepted by a UTCA within time  $\log |\sigma|$ . This improves a result in [1].

#### 4. A comparison with Lindenmayer systems

The language classes of the CHOMSKY hierarchy do not seem to be especially appropriate as UTCA language classes.

Example 2.  $L = \{a^m b^m a^m b^m \mid 4m \text{ is a power of } 2\}$  is not a context-free language. Analogously to Example 1, this language can be accepted by a UTCA within time  $\log |\sigma|$ . For this, we use the following state transition function:

<u>left son's state</u>	<u>right son's state</u>	<u>next state</u>
$\sqcup$	$\sqcup$	$\sqcup$
a	a	a
b	b	b
a	b	t'
t'	t'	t
all other combinations of states		f

Only t is an accepting state.

Example 3.  $L = \{ww^R \mid w \in \{a,b\}^*\}$  is a context-free language.

According to [1], no UTCA can accept L in less than  $|\sigma|/2$  time.

A better approach seems to be to compare UTCA language classes with the languages of LINDENMAYER systems (see, e.g., [2]).

We will show that the class UTCA - TIME ( $\log |\sigma|$ ) can be characterized through a special class of L-systems.

A deterministic power-of-two EOL system (DPTEOL) S is defined as a 4-tuple  $\langle \Sigma, P, A, B \rangle$ , where  $\Sigma$  is a finite, nonempty alphabet,  $B \subseteq \Sigma$  is the basic alphabet,  $A \subseteq \Sigma$  are the axioms, and P is a set of productions  $a \rightarrow bc$ , for  $a, b, c \in \Sigma$ , satisfying the following two conditions:

(a) for all  $a \in \Sigma$  there exists a production  $a \rightarrow bc$  in  $P$ ,  
for some  $b, c \in \Sigma$ ;

(b) for all  $a, b, c, d \in \Sigma$ , if  $a \rightarrow cd$  and  $b \rightarrow cd$  are productions  
in  $P$  then  $a = b$ .

Such a system generates a language  $L \subseteq B^*$ . As a first step,  
we can apply productions in  $P$  to the axioms in  $A$ . If we have  
reached a word  $a_1 a_2 \dots a_m \in \Sigma^*$  at step  $k$ , at step  $k+1$  we apply  
productions in  $P$  simultaneously to all symbols  $a_i$ ,  $i = 1, 2, \dots, m$ .  
A string  $\sigma$  lies in  $L$  iff  $\sigma \in B^*$ , and  $\sigma$  can be generated by this  
process.

Example 4. Let  $S = \langle \Sigma, P, \{1\}, B \rangle$ , where  $\Sigma = \{a, b, 0, 1, x, y\}$ ,  
 $B = \{a, b\}$ , and  $P = \{1 \rightarrow 10, 01, 11, ab, ba, bb; 0 \rightarrow 00, aa; a \rightarrow xy;$   
 $b \rightarrow yx; x \rightarrow xx; y \rightarrow yy\}$ . This system generates the language  
 $L = \{w \in \{a, b\}^* \mid |w| \text{ is a power of } 2, |w| \geq 2, \text{ and in } w \text{ there is}$   
 $\text{one } b \text{ at least}\}$ .

Let  $\text{DPTEOL}_k$  be the class of DPTEOL systems with  $k$  axioms  
at most, for  $k \geq 1$ . Let  $\text{UTCA}_k$  be the class of UTCA's with  $k$   
final states at most, for  $k \geq 1$ . Furthermore,  $L(\text{DPTEOL}_k)$   
denotes the class of all languages which can be generated by  
a system in  $\text{DPTEOL}_k$ .

Theorem 3. (a)  $L(\text{DPTEOL}_k) = \text{UTCA}_k - \text{TIME}(\log |\sigma|)$ , for all  
 $k \geq 1$ .

(b)  $\text{UTCA}_k - \text{TIME}(\log |\sigma|) \subsetneq \text{UTCA}_{k+1} - \text{TIME}(\log |\sigma|)$ ,  
for all  $k \geq 1$ .

(c)  $L(\text{DPTEOL}) = \text{UTCA} - \text{TIME}(\log |\sigma|)$ .

Proof. (a)  $\Leftarrow$ : Let  $S = \langle \Sigma, P, \{a_1, \dots, a_k\}, B \rangle$  be a DPTEOL system. We built up an SUTCA  $A = \langle \Sigma U \{\sqcup, \#\}, BU \{\sqcup, \#\}, \{a_1, \dots, a_k\}, \delta \rangle$  which accepts those languages generated by  $S$ :

$$\delta(a, b) = c \quad \text{if } c \rightarrow ab \text{ is in } P, \text{ and}$$

$$\delta(a, b) = \sqcup \quad \text{if } c \rightarrow ab \text{ is not in } P,$$

for  $a, b \in \Sigma U \{\sqcup, \#\}$ .

Let  $L_1^\ell$  be the set of all strings in  $\Sigma^*$  with length  $2^\ell$  which can be generated by  $S$ , and let  $L_2^\ell$  be the set of all strings in  $(\Sigma U \{\sqcup\})^*$  with length  $2^\ell$  which can be accepted by  $A$ , i.e., we regard all such strings as input strings for  $A$ . For  $\ell = 0$  we have  $L_1^0 = L_2^0 = \{a_1, a_2, \dots, a_k\}$ . Let  $\ell \geq 1$ . Then,

$$\sigma \in L_1^\ell \quad \text{iff } \sigma = b_1 c_1 b_2 c_2 \dots b_m c_m \text{ with } m = 2^{\ell-1}, d_i \rightarrow b_i c_i \text{ in } P, \text{ for } i = 1, \dots, m, \text{ and } d_1 d_2 \dots d_m \text{ in } L_1^{\ell-1}$$

$$\text{iff } \sigma = b_1 c_1 b_2 c_2 \dots b_m c_m \text{ with } m = 2^{\ell-1}, \delta(b_i, c_i) = d_i, \text{ for } i = 1, \dots, m, \text{ and } d_1 d_2 \dots d_m \text{ in } L_2^{\ell-1}$$

$$\text{iff } \sigma \in L_2^\ell.$$

Thus,  $\sigma$  can be generated by  $S$  iff  $\sigma$  can be accepted by  $A$ , for all  $\sigma \in B^*$ .

$\Rightarrow$ : Let  $A = \langle S, I, T, \delta \rangle$  be an SUTCA with  $F = \{s_1, s_2, \dots, s_k\}$ .

We build up a DPTEOL system  $S = \langle \Sigma U \{x_1, \dots, x_m\}, P, \{s_1, \dots, s_k\}, I - \{\sqcup, \#\} \rangle$  which generates those languages accepted by  $A$ :

$$z_1 \rightarrow z_2 z_3 \text{ in } P \text{ if } \delta(z_2, z_3) = z_1, \text{ and for all } z \in \Sigma U \{x_1, \dots, x_m\},$$

for which according to this rule a production  $z \rightarrow z' z''$  is

not yet in  $P$ , we take the productions  $z \rightarrow x_1 x_2, x_1 \rightarrow x_1 x_1,$

$x_2 \rightarrow x_2 x_2$  in  $P$  (cp. Example 4).

Analogously to the first part of this proof using sets  $L_1^\ell, L_2^\ell$  we can prove that  $S$  generates the same language that  $A$  accepts.

(b) For  $k \geq 1$ , we use  $L = \{ab, abab, \dots, (ab)^{k+1}\}$ . This finite language lies in  $UTCA_{k+1} - TIME(\log |\sigma|)$ , but not in  $UTCA_k - TIME(\log |\sigma|)$ .

Let  $\delta(a, b) = t_1$ . Then it follows that  $\delta(t_1, t_1) = t_2$ ,  $\delta(t_2, t_2) = t_3, \dots, \delta(t_k, t_k) = t_{k+1}$ , where  $F = \{t_1, t_2, \dots, t_{k+1}\}$ , and  $t_1, t_2, \dots, t_{k+1}$  are pairwise different. Otherwise, the infinite language  $L = \{(ab)^\ell \mid \ell \geq 1\}$  would be accepted.

(c) This follows immediately from (a).  $\square$

As a last remark, it can easily be proved that the class  $UTCA - TIME(\log |\sigma|)$  is closed under union, intersection, and complement.

## References

- [1] C. R. DYER: Augmented Cellular Automata for Image Analysis. Dissertation, University of Maryland, College Park, 1979.
- [2] G. ROZENBERG, A. SALOMAA (Eds.): L Systems. Lecture Notes in Computer Science 15 (1974), Springer Verlag.

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